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## 5

# Steady flow in pipe networks

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### 5.1 Water pipe networks

A pipe network can be described as a system of interconnected pipes, forming one or more closed loops. A loop may be defined as a connected set of pipes and their end nodes, every node of which is an end-node of exactly two pipes of the set. The term 'node' is applied to any point at which water enters or leaves the network or to any pipe junction within the network. Thus each pipe is defined by a pair of end-nodes.

The distribution of steady flow within such a pipe system is determined by the following factors:

- (1) the head-discharge relationship for each pipe
- (2) the governing network flow equations
- (3) the boundary conditions of the system

### 5.2 Head-discharge relationships for pipes

The head loss  $H$  in a pipe of given length can be related to the discharge  $Q$  by a correlation of the general form:

$$h = r Q^m \quad (5.1)$$

where  $r$  is a pipe resistance coefficient and the exponent  $m$  is a constant. The values to be assigned to  $r$  and  $m$  depend on the flow equation being used.

The preferred flow equation for pipes flowing full, as discussed in Chapter 3, is the Darcy-Weisbach equation (3.23):

$$h = \frac{fLv^2}{2gD}$$

where the friction factor  $f$  is a function of Reynolds number and pipe relative roughness, as defined by the Colebrook-White equation (3.25). For hand calculation the appropriate value of  $f$  may be read from tabulated value sets for pipes. The program PIPFLO (Chapter 3) uses an iterative procedure for the computation of  $f$ .

The Hazen-Williams equation (3.27) is also widely used for water network computation purposes. It can be written in the form:

$$h = 6.818C^{-1.852} L v^{1.852} D^{-1.167} \quad (5.2)$$

When expressed in the form of equation (5.1), these equations become:

$$\text{Darcy-Weisbach} \quad h = \frac{8fL}{\pi^2 g D^5} Q^2 \quad (5.3)$$

$$\text{Hazen-Williams} \quad h = \frac{10.704L}{C^{1.852} D^{4.871}} Q^{1.852} \quad (5.4)$$

where  $H$  is expressed in metres and  $Q$  in  $\text{ms}^{-1}$ .

### 5.3 Network analysis

In any pipe network the number of unknown flows corresponds to the number of pipes in the network and their evaluation involves the solution of an equal number of simultaneous equations. These governing equations are of two types:

1. Continuity equations: the algebraic sum of the flows at any node must be zero (flows into and away from each node must balance).
2. Loop equations: the integrated head loss around any loop must be zero.

Consider a pipe network having  $P$  pipes and  $N$  nodes. The continuity or node equations are of the general form:

$$\sum Q_{ij} + E_i = 0 \quad \text{for any node } i$$

where  $Q_{ij}$  refers to flow from node  $i$  to node  $j$ , the subscript  $j$  representing nodes connected to node  $i$ .  $E_i$  is the external supply/demand at node  $i$ . The sign convention adopted in this text is that flow towards a node is considered as positive, flow away from a node is assigned a negative value. The maximum number of independent node equations is  $(N-1)$ .

The loop equations are of the general form:

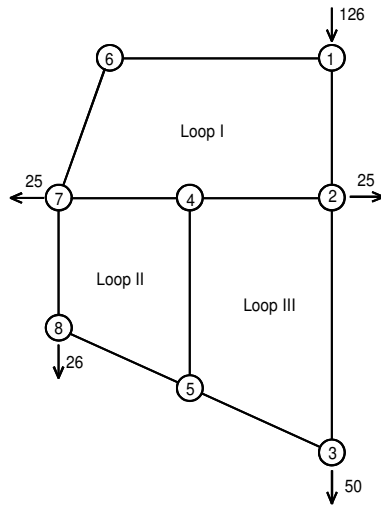
$$\sum h_{ij} = 0 \quad \text{for each loop}$$

where  $h_{ij} = r_{ij}Q_{ij}^m$  is the head loss in the pipe connecting nodes  $i$  and  $j$  and the summation covers all pipes which comprise the loop. The sign convention adopted here is that head loss associated with clockwise flow is considered positive and head loss associated with anticlockwise flow is considered negative. The maximum number of independent loop equations is  $(P-(N-1))$ . Thus the total number of independent equations, derived from the continuity and loop conditions, equals  $P$ , the number of pipes in the network.

The network equations may be expressed in terms of the flow variable  $Q$ , as above, or in terms of node piezometric head  $H$ , using the correlation:

$$h_{ij} = H_i - H_j = r_{ij}Q_{ij}^m$$

The simple network shown on Fig 5.1 has 10 pipes and 8 nodes and the supply and demand values are set out in Table 5.1. Its network equations are therefore made up of 7 node equations and 3 loop equations, which are set out in Tables 5.2 and 5.3. It may be noted that the node equation for node 8 is not an additional independent node equation since it could have been obtained by taking the negative sum of node equations 1 to 7, inclusive. It should be noted that the set of three loops, through an independent set, is not a unique set; any other independent set might have been chosen.



**Fig 5.1** **Three-loop pipe network**  
Indicated supply and demand values in lps

Table 5.1  
Pipe data for network shown in Fig 5.1

Pipe	Length (m)	Diameter (mm)	C-value
1-2	300	300	100
2-3	450	300	100
2-4	150	200	100
3-5	150	180	100
4-5	360	180	100
4-7	150	200	100
5-8	150	180	100
1-6	240	200	100
6-7	300	180	100
7-8	300	180	100

Table 5.2  
Node equations

Node	Equation
1	$126 + Q_{12} + Q_{16} = 0$
2	$-25 + Q_{21} + Q_{23} + Q_{24} = 0$
3	$-50 + Q_{35} + Q_{32} = 0$
4	$Q_{45} + Q_{47} + Q_{42} = 0$
5	$Q_{58} + Q_{54} + Q_{53} = 0$
6	$Q_{67} + Q_{61} = 0$
7	$-25 + Q_{76} + Q_{74} + Q_{78} = 0$

Table 5.3  
Loop equations

Loop	Equation
1	$h_{12} + h_{24} + h_{47} + h_{76} + h_{61} = 0$
2	$h_{23} + h_{35} + h_{54} + h_{42} = 0$
3	$H_{45} + h_{58} + h_{87} + h_{74} = 0$

## 5.4 Boundary conditions

The boundary conditions must be sufficient to define flow distribution. Boundary conditions for water networks include supplies (inflows to the system) and demands (outflows from the system), nodes having constant head e.g. service reservoirs, flow control devices such as non-return valves, pressure reducing valves and pumps.

Typical examples of sets of boundary conditions sufficient to define flow and pressure distribution in water networks are:

- (1) magnitude of supplies and demands are known; one nodal pressure is known;
- (2) magnitudes of pressures at supply nodes and magnitudes of demands are known.

## 5.5 Solution of network equations

As already outlined, the network equations consist of a set of N-1 node equations which are linear in Q and a set of P-(N-1) loop equations which are linear in H. When the latter are expressed in terms of Q, they convert to a set of non-linear equations of the general form:

$$\sum r_{ij}Q_{ij} = 0$$

Alternatively, if the node piezometric head H is taken as the computational variable instead of Q, the network equations convert to a set of P-(N-1) linear equations in H and a set of N-1 nonlinear node equations in H. Since Q is almost invariably the computational variable used in practice, the following discussion is confined to the solution of the network equations expressed in terms of Q.

Direct solution of systems of non-linear simultaneous equations is not feasible, hence it necessary to use iterative solution methods. In general, these methods start with an estimated solution which is iteratively refined by repeated corrections until the deviation from the true solution is reduced to an acceptable tolerance value. Three such iterative solution techniques, which have been applied to pipe network analysis, are reviewed. This review is followed by a manually worked illustrative example using the Hardy Cross method of analysis. The latter method is also used as the basis of the computer program PNA, which analyses flow in pipe networks. The selected three iterative computational procedures are based on:

- (1) Loop by loop flow correction (Hardy Cross)
- (2) Simultaneous loop flow correction
- (3) Linearisation of the network loop equations

### 5.5.1 Hardy Cross method

The most widely used technique for water network analysis is based on a loop by loop iterative computational procedure first described by Hardy cross (1936). Its use may be illustrated by reference to the 3-loop network in Fig 5.1. As already shown, the set of network equations for this network comprises 7 node equations and 3 loop equations. As a first step, an arbitrary flow distribution, which satisfies flow continuity at nodes, that is, complies with the node equations, is assumed. Inevitably, this assumed flow distribution would not satisfy the loop equations. Let the out-of-balance head in loop 1 be  $h_1$ , where

$$h_1 = r_{12}Q_{12}^m + r_{24}Q_{24}^m + r_{47}Q_{47}^m + r_{76}Q_{76}^m - r_{61}Q_{61}^m$$

The first order estimate of the loop flow correction  $\Delta q_1$ , which would reduce  $h_1$  to zero, is found by the Newton-Raphson approximation (described in Appendix B):

$$-h_1 = \frac{dh_1}{dq_1} \Delta q_1 \quad (5.5)$$

where the numerical values of  $h_1$  and its differential are based on the current  $Q$  values in loop 1 pipes. The loop flow correction  $\Delta q_1$  is, therefore

$$\text{Loop 1:} \quad \Delta q_1 = -\frac{h_1}{dh_1 / dq_1}$$

On the same basis, the loop flow corrections for loops 2 and 3 are

$$\text{Loop 2:} \quad \Delta q_2 = -\frac{h_2}{dh_2 / dq_2}$$

$$\text{Loop 3:} \quad \Delta q_3 = -\frac{h_3}{dh_3 / dq_3}$$

Because these flow corrections are loop flows i.e. clockwise or anticlockwise flows applied to all pipes in the loop, they do not invalidate the initially satisfied nodal continuity equations. Flow corrections are repeated on a loop by loop basis until the maximum out-of-balance loop head is reduced to a specified tolerance value.

### 5.5.2 Simultaneous loop flow correction

The Newton-Raphson method has also been used to compute simultaneous flow corrections for all loops (Martin and Peters, 1963; Epp and Fowler, 1970). This computational procedure takes into account the interactive influence of flow corrections between loops which have common pipes. As done in the Hardy Cross method, an initial flow distribution, which satisfies flow continuity at nodes, is assumed. The network of Fig 5.1 is again used as an illustrative example. The first order estimates of the loop flow corrections, which would reduce the loop out-of-balance heads to zero, are found from the following Newton-Raphson approximations:

$$-h_1 = \frac{\partial h_1}{\partial q_1} \Delta q_1 + \frac{\partial h_1}{\partial q_2} \Delta q_2 + \frac{\partial h_1}{\partial q_3} \Delta q_3$$

$$-h_2 = \frac{\partial h_2}{\partial q_1} \Delta q_1 + \frac{\partial h_2}{\partial q_2} \Delta q_2 + \frac{\partial h_2}{\partial q_3} \Delta q_3$$

$$-h_3 = \frac{\partial h_3}{\partial q_1} \Delta q_1 + \frac{\partial h_3}{\partial q_2} \Delta q_2 + \frac{\partial h_3}{\partial q_3} \Delta q_3$$

This set of linear simultaneous equations in  $\Delta q$  can be written in matrix form as follows:

$$\begin{bmatrix} \frac{\partial h_1}{\partial q_1} & \frac{\partial h_1}{\partial q_2} & \frac{\partial h_1}{\partial q_3} \\ \frac{\partial h_2}{\partial q_1} & \frac{\partial h_2}{\partial q_2} & \frac{\partial h_2}{\partial q_3} \\ \frac{\partial h_3}{\partial q_1} & \frac{\partial h_3}{\partial q_2} & \frac{\partial h_3}{\partial q_3} \end{bmatrix} \begin{bmatrix} \Delta q_1 \\ \Delta q_2 \\ \Delta q_3 \end{bmatrix} = \begin{bmatrix} -h_1 \\ -h_2 \\ -h_3 \end{bmatrix}$$

This set of linear simultaneous equations is conveniently solved by matrix inversion, thus yielding the required set of loop flow corrections,  $\Delta q_1$ ,  $\Delta q_2$  and  $\Delta q_3$ .

The illustrative example used had three loops, resulting in a 3x3 matrix of differentials. Thus, a network having L loops would result in an LxL matrix of differentials. It is clear that the matrix elements will have non-zero values only where they connect loops having common pipes, that is,  $dh/dq_y$  will only have a non-zero value if loops x and y share one or more common pipes.

It is reasonable to expect that the simultaneous loop flow correction method should converge more rapidly to the true solution than the loop by loop Hardy Cross method.

### 5.5.3 Linearisation of network equations

Wood and Charles (1972) outlined a linearisation of the loop equations, thus enabling the direct solution of the full set of network equations. Each loop equation is of the general form

$$\sum r_{ij} Q_{ij}^m = 0$$

which can be written in the modified form

$$\sum r_{ij} Q_{ij}^{m-1} Q_{ij} = 0$$

or

$$\sum \bar{r}_{ij} Q_{ij} = 0$$

where  $\bar{r}_{ij} = r_{ij} Q_{ij}^{m-1}$  is treated as a numerical coefficient based on the current value of  $Q_{ij}$ .

To start this method it is also necessary to assume an initial flow distribution. After each solution of the full set of linearised network equations, a new set of  $\bar{r}$  pipe coefficients is calculated for use in the next iteration.

### 5.5.4 Convergence of methods

It has been demonstrated (Wood and Charles, 1972) that, as might be expected, of the foregoing methods of analysis, the Hardy Cross method provides the least rapid convergence. This is because each loop flow correction is made in isolation from the rest of the network, whereas the other two methods incorporate full network interaction in each iteration.

The Hardy Cross method, however, is not to be dismissed as a useful method of network analysis as the computer time required may not be of great economic significance in the overall task of network analysis and design. The method has the advantage over the other methods of requiring less computer memory capacity as it does not involve the solution of large sets of linear simultaneous equations.

Convergence of all iterative methods is aided by judicious selection of the initial flow distribution, as discussed in section 5.7.

## 5.6 Worked example: Hardy Cross

The calculation procedure for the computation of the flow distribution for the simple 3-loop pipe network shown in Fig 5.1, based on the Hardy Cross computation method, is outlined in Table 5.4. The first column of  $Q_{ij}$  values shows the assumed initial flow distribution, which, as required, satisfies the node continuity condition at all nodes. The loop flow correction  $\Delta q$  is based on eqn (5.5), which for any loop x can be written as

$$\Delta q_x = - \left( \frac{h}{dh/dq} \right)_x$$

Hence

$$\Delta q_x = - \left( \frac{\sum r_{ij} Q_{ij}^m}{m \sum r_{ij} Q_{ij}^{m-1}} \right)_x$$

or

$$\Delta q_x = - \left( \frac{\sum h_{ij}}{m \sum (h/Q)_{ij}} \right)_x \quad (5.6)$$

In general, the iterative correction process is continued until the integrated head loss around all loops in the network is reduced to a specified limiting value.

## 5.7 Loop selection

As shown earlier, the number of independent loop equations for a network is  $P-(N-1)$ . Even in simple networks such as that shown in Fig 5.1, the total number of possible loops is greatly in excess of the number of independent loops, which for this network is 3, as shown in Table 5.5. It will be evident that the loop equation based on loop 4 could be obtained by combining the loop equations for loops 1, 2 and 3, while the loop equation for loop 5 could be obtained by combining the loop equations for loops 2 and 3, and so on.

For simple networks, such as that in Fig 5.1, a set of independent loops is immediately obvious by inspection. This is not usually the case for more complex networks and hence a defined procedure is required for selecting an independent set of loops. The optimal set of independent loops is that set which leads to the most rapid rate of convergence of the iterative calculation process.

It has been shown (Travers, 1967) that convergence is most rapid if the chosen set of independent loops is comprised of those loops for which the summed resistances ( $r$ -values) of their constituent pipes are minimised. Obviously, such loops are based on the low resistance pipes of the system i.e. the low resistance pipes are included in as many loops as possible while the high resistance pipes are included in as few loops as possible.

Table 5.5

		$\Sigma r$
Loop 1	P <sub>12</sub> P <sub>24</sub> P <sub>47</sub> P <sub>76</sub> P <sub>61</sub>	5817.5
Loop 2	P <sub>23</sub> P <sub>35</sub> P <sub>54</sub> P <sub>42</sub>	5719.5
Loop 3	P <sub>45</sub> P <sub>58</sub> P <sub>87</sub> P <sub>74</sub>	8077.0
Loop 4	P <sub>12</sub> P <sub>23</sub> P <sub>35</sub> P <sub>58</sub> P <sub>87</sub> P <sub>76</sub> P <sub>61</sub>	9927.7
Loop 5	P <sub>23</sub> P <sub>35</sub> P <sub>58</sub> P <sub>87</sub> P <sub>74</sub> P <sub>42</sub>	7333.5
Loop 6	P <sub>12</sub> P <sub>23</sub> P <sub>35</sub> P <sub>54</sub> P <sub>47</sub> P <sub>76</sub> P <sub>61</sub>	9925.7
Loop 7	P <sub>12</sub> P <sub>24</sub> P <sub>45</sub> P <sub>58</sub> P <sub>87</sub> P <sub>76</sub> P <sub>61</sub>	12283.0

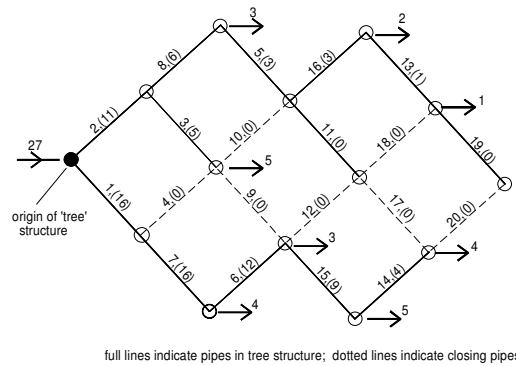
Table 5.4  
Analysis of network shown in Fig 5.1 by iterative solution of the loop equations

Loop	Pipe: i-j	$r_{ij}$	$Q_{ij}$	$H_{ij}$	$H_{ij}/Q_{ij}$	$\Delta Q$	$\Delta Q$	$Q_{ij}$	$H_{ij}$	$H_{ij}/Q_{ij}$	$\Delta Q$	$\Delta Q$
I	1-2	223.7	+0.090	+2.600	28.89	+0.011	+0.000	+0.101	+3.218	31.85	-0.000	+0.000
	2-4	806.0	+0.010	+0.161	16.10	+0.011	+0.007	+0.028	+1.080	38.57	-0.000	-0.004
	4-7	806.0	+0.005	+0.045	9.00	+0.011	-0.003	+0.013	+0.261	20.08	-0.000	+0.002
	7-6	2693.0	-0.036	-5.746	159.61	+0.011		-0.025	-2.927	117.08	-0.000	
	6-1	1289.5	-0.036	-2.752	76.44	+0.011		-0.025	-1.402	56.08	-0.000	
			$\Sigma$	-5.692	290.04			$\Sigma$	0.230	263.67		
			$\Delta Q = 5.692/(1.85 \times 290.04) = 0.011$				$\Delta Q = -0.230/(1.85 \times 263.67) = -0.000$					
II	2-3	335.5	+0.055	+1.568	28.51	-0.007		+0.048	+1.219	25.40	0.004	
	3-5	1346.5	+0.005	+0.075	15.00	-0.007		-0.002	-0.014	7.00	0.004	
	5-4	3231.5	-0.005	-0.179	35.80	-0.007	-0.003	-0.015	-1.365	91.00	0.004	0.002
	4-2	806.0	-0.010	-0.161	16.10	-0.007	-0.003	-0.028	-1.080	38.57	0.004	0.002
			$\Sigma$	+1.303	95.41			$\Sigma$	-1.240	161.97		
			$\Delta Q = -1.303/(1.85 \times 95.41) = -0.007$				$\Delta Q = 1.240/(1.85 \times 161.97) = 0.004$					
III	4-5	3231.5	+0.005	+0.179	35.80	0.003	0.007	+0.015	+1.356	91.00	-0.002	-0.004
	5-8	1346.5	+0.010	+0.269	26.90	0.003		+0.013	+0.437	33.62	-0.002	
	8-7	2693.0	-0.016	-1.282	80.13	0.003		-0.013	-0.873	67.1	-0.002	
	7-4	806.0	-0.005	-0.045	9.00	0.003	-0.011	-0.013	-0.261	20.08	-0.002	+0.000
			$\Sigma$	-0.879	151.83			$\Sigma$	+0.668	211.85		
			$\Delta Q = 0.879/(1.85 \times 151.83) = 0.003$				$\Delta Q = -0.668/(1.85 \times 211.85) = -0.002$					

Ideally, therefore, a loop selection procedure should be able to define the optimal set of independent loops so as to reduce computational time to a minimum. The following loop selection algorithm, described by Travers (1967), approximately satisfies this requirement. The method includes the following steps, as illustrated in Fig 5.2.

- (1) All pipes in the system are ranked in order of increasing resistance (r-value), the pipe of lowest resistance becoming pipe number 1, the pipe of next lowest resistance becoming pipe number 2 and so on.
- (2) A **tree** structure is constructed within the network, consisting of the pipes taken in order from the ranked list. A tree consists of a connected set of pipes and their end nodes, no subset of which comprises a loop. When this procedure is complete, all N nodes of the network will be included and the number of pipes forming the tree structure will be N-1. If the total number of pipes is P, then the number of non-tree pipes is P-(N-1). These non-tree pipes form the closing pipes of the required set of independent loops.
- (3) When the tree structure is complete there is then a unique path from each node back to the origin of the tree (one of the end nodes of the pipe of lowest resistance). The loop designated by any of the closing pipes is found as follows. The end nodes of each closing pipe are assigned a level number which corresponds to the number of pipes in the path from that node back to the origin. Taking, as an example, a particular closing pipe with end nodes a and b and, starting from the node of higher level (node a), proceed towards the origin of the tree until a node having the same level as b is reached. This node may in fact coincide with node b, in which case the required loop is defined. If this is not the case, then proceed along both paths towards the origin, adding in successive pipes in each path, in turn, until the paths cross before or at the origin, and a closed loop is formed.

This loop selection procedure is outlined in Fig 5.2 and Table 5.6.



**Fig 5.2**

**Loop selection and flow initialisation**  
**x, (y) are pipe rank and initial flow, respectively.**

Table 5.6  
 Selected set of loops

Loop number	Pipes
1	4, 1, 2, 3,
2	10, 3, 8, 5
3	9, 3, 2, 1, 7, 6
4	12, 6, 7, 1, 2, 8, 5, 11
5	17, 14, 15, 6, 7, 1, 2, 8, 5, 11
6	18, 11, 16, 13
7	20, 14, 15, 6, 7, 1, 2, 8, 5, 16, 13, 19

## 5.8 Initial flow distribution

Before the iterative process can be started, an initial distribution of flow must be assumed. Where supplies and demands are specified, this initial flow distribution is uniquely defined by the network tree if all closing pipes are assumed to have zero flow. Since the pipes in the tree are mainly the lower resistance pipes, this initial assignment of flows may be assumed to be nearer the final solution than an arbitrarily assumed distribution. An example of the above procedure is shown in Fig 5.2.

In cases where fixed heads are specified rather than supply values, a set of initial supply values, which satisfies flow continuity conditions, must be assumed (this problem is further discussed in the following section).

## 5.9 Network flow controls

Flow controls in a water network may include non-return valves, pressure-reducing valves, pumps and points of fixed pressure such as service reservoirs. The latter are always located at nodes while the remainder are non-nodal in location.

### *Non-return valves*

If the computational procedure reveals a backward flow through a non-return valve, a correction can be made by introducing an equal and opposite loop flow or alternatively the network can be re-analysed with the pipe containing the non-return valve omitted.

### *Flow-regulating valves*

Such devices cause a step-change in head  $H$ , which can usually be related to the flow  $Q$  by an equation of the form:

$$h_v = K_v Q^2 \quad (5.7)$$

where  $K_v$  is a variable coefficient whose value is a function of the extent of valve opening. ( $K_v$ -values may be calculated from the valve  $K$ -values given in Table 3.5).

The flow correction  $\Delta Q$  for a loop containing a valve is found by adding the appropriate valve terms to eqn (5.6):

$$\Delta Q = - \frac{\sum h_{ij} + h_v}{m \sum \left( \frac{h}{Q} \right)_{ij} + 2K_v Q} \quad (5.8)$$

### *Pumps*

Pumps cause a step-increase in head in the direction of flow, which can generally be determined from a pump characteristic equation of the form

$$h_p = A_0 + A_1 Q + A_2 Q^2 \quad (5.9)$$

where  $A_0$  is the 'shut-off' head, and  $A_1$  and  $A_2$  are constant coefficients.

The flow correction  $\Delta Q$  for a loop containing a pump is found by adding the appropriate pump terms to eqn (5.6):

$$\Delta Q = - \frac{\sum h_{ij} + h_p}{m \sum \left( \frac{h}{Q} \right)_{ij} + (2A_2 Q + A_1)} \quad (5.10)$$

(Note: for the sign convention which associates positive h values with clockwise loop flow, h has a positive value if the pumping direction is anticlockwise.)

### **Points of fixed head**

In network problems where supplies and demands are specified, the nodal pressure distribution can be found provided a single nodal pressure is known. Each additional specified node head constitutes an additional boundary condition.

Suppose x and y nodes have specified node heads,  $H_x$  and  $H_y$ , respectively. This boundary condition can be covered by an additional equation of the form

$$\sum h_{ij} = H_x - H_y$$

which, in terms of Q, becomes

$$\sum r_{ij} Q_{ij}^m - (H_x - H_y) = 0 \quad (5.11)$$

where the summation refers to all pipes in the connected path from node x to node y. If this equality is not fulfilled then a correction  $\Delta Q$  can be applied to all pipes in that path, such that

$$\sum r_{ij} (Q_{ij} + \Delta Q)^m - (H_x - H_y) = 0$$

This correction is found by the Newton-Raphson procedure to be

$$\Delta Q = - \frac{\sum h_{ij} - (H_x - H_y)}{m \sum \left( \frac{h}{Q} \right)_{ij}} \quad (5.12)$$

The flow correction  $\Delta Q$  is also applied to the supply/demand values at nodes x and y, resulting in variations from their assumed values. Thus, where more than one nodal head is fixed, the supplies/demands at points of fixed head must be treated as variables in network computation. If the number of nodes of fixed head (must be supply /demand nodes) is  $N_f$ , the required number of additional boundary equations of the type (5.11) is  $N_f - 1$ .

Although equation (5.12) is valid for any connected path between x and y, the most convenient path is that defined by the network tree.

## **5.10 Analysis of existing distribution systems**

Existing water distribution systems may have a large number of interconnected pipes, varying in size from small diameter service pipes to large diameter trunk mains. In order to reduce the analytical task to manageable proportions, it is usually necessary to develop a simplified model of the network, which includes the main arteries of flow. This is conveniently carried out using a scale drawing of the network, on which the main pipe intersections (network nodes) are identified. It will usually be possible to replace minor pipes by equivalent supplies/demands at nodal points. A schematic outline of the simplified network can then be prepared.

The data required for network analysis include:

- (1) pipe data : length, diameter, roughness, node elevations
- (2) supply/demand data

- (3) data on flow regulation: pumps, control valves, service reservoirs etc.
- (4) field data from monitoring flows and heads at selected points in the network. This information is essential to verify that the simplified network is a satisfactory model of the actual system.

The model network is analysed and the results are compared with field data. Discrepancies are noted and the model network is appropriately modified and re-analysed. This procedure is repeated until the computed results are found to satisfactorily accord with the field observations. The schematic model can then be regarded as proven and can be reliably used as an analytical tool for the design of modifications and/or extensions.

### 5.11 Network analysis by computer

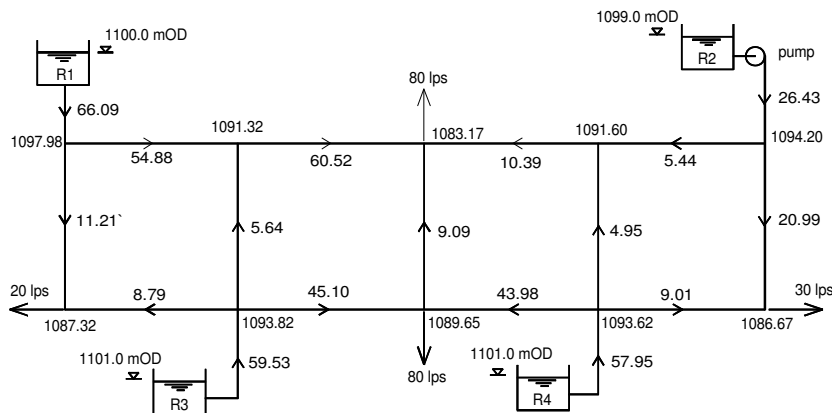
The analysis of flow and pressure distribution in water networks is a tedious and time-consuming task, conveniently carried out by computer. As well as providing a means of analysis, the computer also allows the storage of network information on file, which is of particular benefit in network design where repeated modification and analysis is typically necessary.

The ARTS (Aquavarrá Research, 2000) hydraulic design software package facilitates the graphical representation of pipe networks and associated regulatory controls on screen and enables the user to carry out a steady flow analysis. Control elements catered for include multiple reservoir inputs, pumps, pressure-reducing valves, and non-return valves. The coding uses the Travers algorithm for loop selection and flow initialisation, as previously described; analysis is based on the Hardy Cross-iterative loop flow correction procedure.

#### Sample computation

The network shown on Fig 5.3 is used to illustrate the application of the ARTS software. The user constructs the network on screen by selecting the components from the tool palette and replicating the connectivity of the network of pipes – the components in this instance are pipes, reservoirs, a pump and a set of demands. The properties are then assigned to each component as follows:

Component	Properties
Pipes	length, internal diameter, wall roughness
Pipe junctions	Node elevations (mOD)
Reservoirs	top water level (TWL mOD)
Pumps	head/discharge characteristic (defined by three points on the head/discharge curve)
Demands	value ( $l\ s^{-1}$ )



**Fig 5.3** Pipe network diagram

The analytical results are printed on the network, as illustrated in Fig 5.3, where the flows are in l s-1 and the nodes heads are the potential head values (mOD). The user also has the option of printing the node gauge pressures (m).

The validity of the computed flow and pressure distributions should be manually checked by verifying the consistency of the flow and node head differences for selected individual pipes.

## References

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